RF PROJECT 1444

THE OHIO STATE UNIVERSITY

RESEARCH FOUNDATION

1314 KINNEAR ROAD COLUMBUS, OHIO 43212

FINAL REPORT

THE USE OF PHOTOGRAMMETRIC METHODS TO INVESTIGATE SURFACE MOVEMENT OF THE ANTARCTIC ICE SHEET

National Science Foundation
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The Use of Photogrammetric Methods

To Investigate

Surface Movement of the Antarctic Ice Sheet

by

S. Weissman

Page	
10	3rd line from bottom: For "the "x" axis (φ)", read "(Φ)"
13	To clarify numbering of bases - enclose in parenthesis
16	7th line from top: For "grid syste", read "grid system"
17	3rd equation from bottom: For "$\delta\alpha_E \cdot (A_E - \alpha_E)$", read "$\delta\alpha_E = (A_E - \alpha_E)$"
37	Above the first equation add: "Dr. Brandenberger's empirical formula is:"
37	7th line from bottom: For "a strip of models", read "a strip of 20 models"
38	1st equation from bottom **after** the equal sign: read $\pm$ 14.6 m
46	Title of figure reading "bz" curves: read "bz" curve
49	Underline: Section XII title
50	10th line from top: For "Forest's preliminary report": read "Forrest's preliminary report"
52	In Figure 11 the letter "2" should read "L"
53	1st equation from bottom: For "$A_D = A_N + \delta - 180^\circ$" read "$A_D = A_H + \delta - 180^\circ$"

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To: NATIONAL SCIENCE FOUNDATION
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On: THE USE OF PHOTOGRAMMETRIC METHODS
TO INVESTIGATE
SURFACE MOVEMENT OF THE ANTARCTIC ICE SHEET

For the Period: 31 May 1963 - 1 Sept 1964

Prepared by: Simha Weissman
Research Assistant

Submitted by: Dr. Arthur J. Brandenberger
Project Supervisor

Date: Sept. 1964

FOREWORD

This report was prepared by Mr. Simha Weissman, Research Assistant, Department of Geodetic Science, The Ohio State University, under National Science Foundation Grant G-23006, OSURF Project No. 1444, with Dr. Arthur J. Brandenberger, Professor, Department of Geodetic Science, as Project Supervisor.

The author wishes to acknowledge the assistance of Mr. Afifi H. Soliman, Research Assistant, who helped in preparing this report and to thank Dr. S. K. Ghosh, Assistant Supervisor, for his helpful advice.

Appreciation is also extended to Mrs. Vera N. Hoff, Secretary and to Mr. F. W. Twitty, Technical Assistant, for their assistance.

ABSTRACT

This report represents a part of a study being made to measure the surface movement of the Antarctic ice sheet using photogrammetric methods.

The area under investigation is between Byrd Station and Mt. Chapman. Markers were placed in this area along a distance of 365 km and their positions were determined by aerial triangulation. One hundred twenty eight models were bridged and the coordinates obtained will be compared with future observations in order to determine the rate of movement of the ice sheet.

This report evaluates the work accomplished in the field, explains the performance of the aerial triangulation, analyzes the accuracy obtained and suggests a proposal for future work in view of the experience gained.

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THE USE OF PHOTOGRAMMETRIC METHODS TO INVESTIGATE SURFACE MOVEMENT OF THE ANTARCTIC ICE SHEET

I. Introduction

The determination of the rate of movement of the Antarctic ice sheet is a part of the Antarctic research program under the direction of the National Science Foundation assisted by the United States Navy.

This project, under the Supervision of Dr. A. J. Brandenberger, The Ohio State University, Department of Geodetic Science, and the Institute of Polar Studies, was started in the austral summer 1962-63 by placing approximately 140 permanent markers between Byrd Station and Mt. Chapman in the Whitmore Mountains covering a strip area of approximately 375 km x 8 km. A survey network was established by Mr. Robert B. Forrest, Principal Investigator, and Mr. D. T. Dickson, Research Assistant. The network consists of seven independent base lines, sun azimuths, vertical angle measurements for slope determination, barometric height measurements and horizontal angle measurements between adjacent traverse legs, as well as approximate distance measurements between the targets.

The flight missions were performed by the Navy Air Development Squadron VI along the established line of markers to provide the necessary material for strip triangulation. The southern end of the strip triangulation is tied to ground control on rock at Mt. Chapman and the survey network provides the necessary control between the mountain and Byrd Station. In periods of one to three years the position of the markers will be determined by aerial triangulation and the difference between the coordinates as determined each time will provide the annual rate of displacement of the ice sheet.

The data obtained in the field were computed and the results as well as the detailed description of the work accomplished and the methods used are presented in the preliminary report prepared by Mr. R. B. Forrest [1] to The National Science Foundation.

This report is a continuation of the report mentioned above and it is assumed that the material covered in the above report is known to the reader of this report.

II. Evaluation of the field work

A. The main advantage of using photogrammetric methods to determine the rate of movement of the ice sheet is that it enables measurement of the relative position of the targets on the ground at one specific time and thus reduces considerably the length of time of conventional surveying during which the targets are moving with the ice sheet. Previous experimants indicate quite a fast annual movement of the ice sheet, and therefore the main concern should be to reduce the field work to a minimum. In view of this the field work completed in the austral summer 1962-63 must be examined.

The mere fact that the time spent in the field exceeds one month indicates that the main requirement mentioned above is not entirely fulfilled. Assuming an annual movement of 100 m (and it may be much more than this) we may expect at least a movement of 8 to 10 m during the time the survey took place, and this amount might already be too large.

The establishing of the targets together with the surveying while heading toward Mt. Chapman seems to add unnecessary time to the measurement period. Pre- location of the targets and measuring them later will increase the effort of the workers and at the same time will increase the accuracy by shortening the measuring period.

B. It is not necessary to take solar azimuths, in this particular work, if horizontal angles are measured between the markers and if one "fixed" line exists in the area, nor is it needed to occupy each marker for determining the azimuth of the base lines along the strip. All these savings in field work would serve as a basis for an alternate proposal to perform future field work.

C. The original program requires photo strips taken from two different altitudes to help in identification and in case one of the strips will not be suitable for later work.

Such a program needs a corresponding arrangement of markers on the ground to meet the possibility that one of the strips will be unsuitable. The arrangement of the markers on the ground provides an adequate work only for the high altitude strip (20,000 ft above the ice surface). No absolute orientation can be made at the beginning and end of each leg in case the low altitude strip (10,000 ft above the ice surface) is used.

(See Figure 1 for the missing targets.)

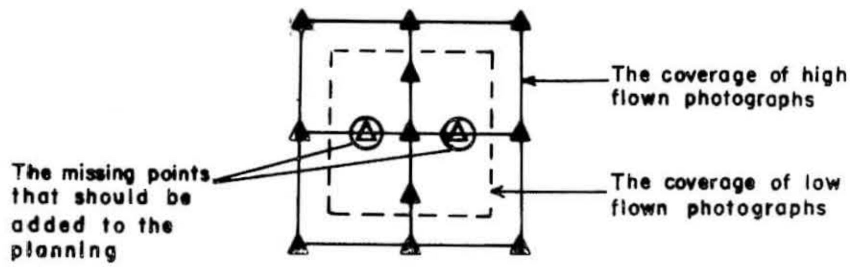

Figure 1

Missing points in planning

Moreover, the low altitude strip cannot help in identifying points on the high altitude strip. The only features available on the photos are the sastrugi ~~crevasses~~, whose shapes are identified by their shadows only. The same sastruga ~~crevasse~~, photographed from a different angle or while the shadows are in different directions, will be seen entirely differently. Since the photo strips were taken from different angles and at different times of day, they cannot be matched.

To sum up: the method used is suitable under normal conditions but is not well adapted to this particular kind of work in the Antarctic area.

III. Evaluation of the photography

The rolls of negatives were examined on the light table. In the high altitude strip the markers, as well as other ground structural features, were not visible, due mainly to the very poor resolution of the taking camera. A sample of these negatives were printed and examined further under a mirror stereoscope and in the Wild Autograph A7. These examinations led to the final conclusions that the high altitude strip cannot be used for triangulation purposes.

The other two strips taken from low altitude were next examined on the light table, while observing the principal targets (principal target = a target combined of 4 markers, located between each two legs and at the beginning and end of the strip).

In one strip all of the principal targets could not be observed and it must be assumed that the strip has a parabolic shaped flight axis as illustrated in Figure 2.

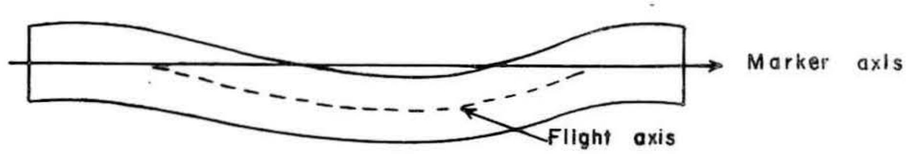

Figure 2

Shape of the disregarded low altitude strip

The other strip includes all the principal targets and its quality seemed to be the best in comparison with the other strip. For this strip paper prints were made and a closer examination could be performed.

In each model two homologous features were identified (this part of the work was the most difficult one) and brought under the mirror stereoscope, thus facilitating stereoscopic vision. The edges of each model were then drawn on the prints and the overlaps were thus obtained.

The principal targets were easily visible by naked eye, but all other single markers could not be identified immediately. In order to locate them each leg was laid out according to the overlaps drawn previously and the distance between the two extreme principal targets was measured and compared with the given approximate distance. Knowing the approximate photo scale, the scaled distances between the single markers were measured along the line connecting the two principal targets. In this way the approximate location of each of the targets were obtained and examined further until the targets were identified with sufficient certainty.

Having the above data at hand, the following conclusions are evident:

A) The poor resolving power of the camera is the main source of troubles, causing uselessness of the high altitude strip and the following inconveniences:

1) The use of the low altitude strip doubles the amount of models to be bridged, thus, longer time in preparation and performance of the work, increased effort and decreased accuracy are unavoidable.

2) Several lateral targets (at the edges of the coverage) are not covered by the low altitude strip.

3) The unfavorable arrangement of the control points for the low altitude strip (as explained previously) and the effect of the poor resolution on the accuracy of the relative orientation (as explained later) reduced the accuracy of the triangulation procedure.

B) The poor mechanical quality of the camera made it impossible to regulate the overlap and as a result, besides having a different percentage of overlap between the models, there is a gap in the middle of the whole strip which cannot be bridged. The following data demonstrates clearly the above statement.

Leg	Photo No.	Overlap
1	303-252	70-80 %
2	252-243	65-70 %
	243-214	75-80 %
	214-195	80-85 %
3	195-188	85 %
	187-185	75 %
	185-177	80-85 %
	177-176	25 %-(gap)
	176-175	80 %
	175-174	No ground coverage for 6000 m (large gap)
	174-160	75 %
4	160-115	65 %
5	115-072	60-70 %
6	072-043	65 %
	043-023	85 %

C) In spite of the difficulties of navigation in this area, the navigation was relatively good except for Legs 5 and 6.

In Leg 5 the flight does not follow the established line and as a result one ground control point at the last array is missing. (See Figure 3.)

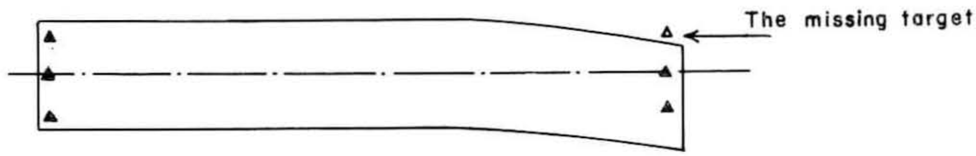

Figure 3

Missing target in leg 5

In Leg 6 an attempt was made to return to the required heading but it was not done gradually and the result is a jagged strip as illustrated in Figure 4.

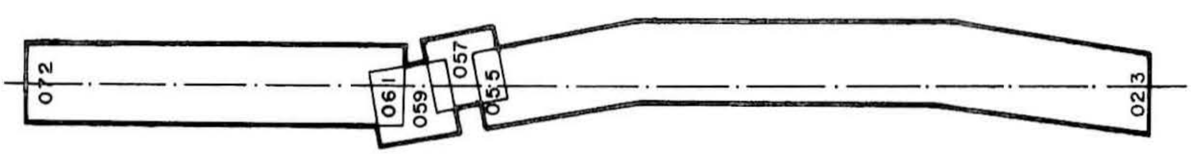

Figure 4

Shape of leg 6

IV. Preparations for the instrumental work

Nine circles were marked on each photograph in the triple overlap areas. This predetermination assures the process and continuation of the triangulation and will help to locate homologous points during the relative orientation procedure.

Negatives were used in performing the aerial triangulation instead of glass diapositives for economical reasons. The negative roll was cut and each photo was taped to the photo holder and flattened by a glass placed upon it. This procedure is usually not permitted in first-order work, but in this case it was felt that the other errors involved in the work exceed those resulting from possible film shrinkage and other related errors.

V. Method of aerial triangulation

The aerial polygon method was selected to bridge the independent geodetic control established in the field. This is not the most favorable method for this particular case as will be explained later. The method to be used here would be the aeroleveling, but this method requires known and reliable statoscope data. Since such data was not available, only the aeropolygon method could be selected.

Forming an optical stereoscopic model from the overlap area of the first two photographs, performing the relative orientation, scaling and leveling the model with the help of the data obtained from the field, the position of the two photographs in space thus became fixed, and the coordinates of each desired point on the model can be read directly. Replacing the first photo by the third one and relatively orienting it to the second photograph, the third photograph becomes fixed in space.

Transferring the scale and elevation from the first to the second model, the same system and datum of coordinates are re-established in the second model. The triangulation continues in this manner to the end of the strip.

Since this method is well known, there is no need to enter into details of this procedure and the reader is referred to the special publication written by Dr. A. J. Brandenberger [3] for further details.

VI. Performance of the aerotriangulation

The work was performed on a first-order stereo instrument, the Wild Autograph A7. The model scale was selected to be 1:10 000. The x and y coordinates were read in that scale to the nearest hundredths of a mm, while the elevations were read directly in m (ground elevation). All readings were recorded in the tables prepared for this purpose.

The aerialpolygon method does not permit triangulation of very long strips because the accumulation of systematic errors (including earth curvature effect) eventually exceed the range of the orientation elements at the instrument and results in an unfavorable correlation of these elements. Therefore, the strip was sub-divided into sections each including one leg. Only legs No. 4, 5, and 6 were bridged because of the gap in leg 3.

Since the ground control points in the first model of each leg are arranged in a straight line along the "y" axis and no control point was available along the "x" axis of the model, the leveling of the model was performed along its "y" axis only (Ω) and not along the "x" axis (φ), thus the longitudinal tilt will be the largest factor to be adjusted later.

~~Crevasses~~ sastrugi were chosen for pass points and the floating mark was set on them in a place which can be identified as well as possible, but of course, the standard pointing error could not reach the desired value of 0.01 mm.*

Due to the poor resolution quality of the camera, the relative orientation is somewhat weak, although no y-parallaxes were visible. (Note: absense of evident parallax is not a complete criterion for the precision of relative orientation. When the background is not sufficiently detailed, it is difficult to observe existing y-parallaxes.)

Since the first photograph was set in the instrument having its corresponding orientation elements at their zero positions and since the flight axis was not brought to coincide with the machine axis, the element by was used to its full range several times. This was avoided while triangulating the other legs by giving the first camera a certain initial swing κ.

One hundred twenty eight models were triangulated in three separate strips (see Figures 5 and 6). All the orientation elements as well as the three coordinates of each of the pass points and markers were recorded. Table I represents the average machine coordinates of the markers only.

* As far as the pointing accuracy on the markers are concerned, the pointings on the markers located along the flight axis are generally good. The markers at the corners were blurred and the accuracy obtained there is no better than the accuracy obtained at the pass points.

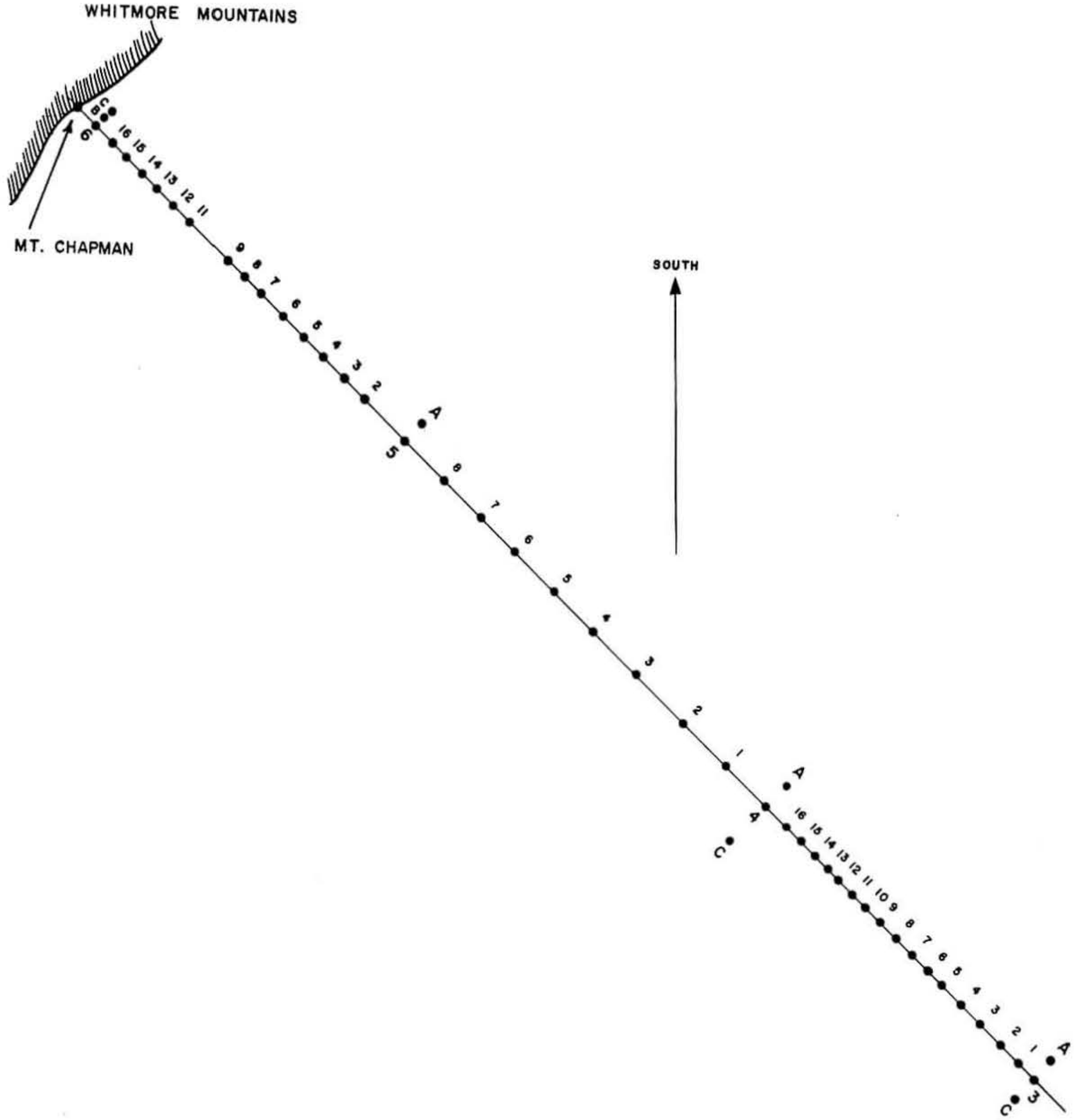

Fig. 5. The path covered by the triangulation

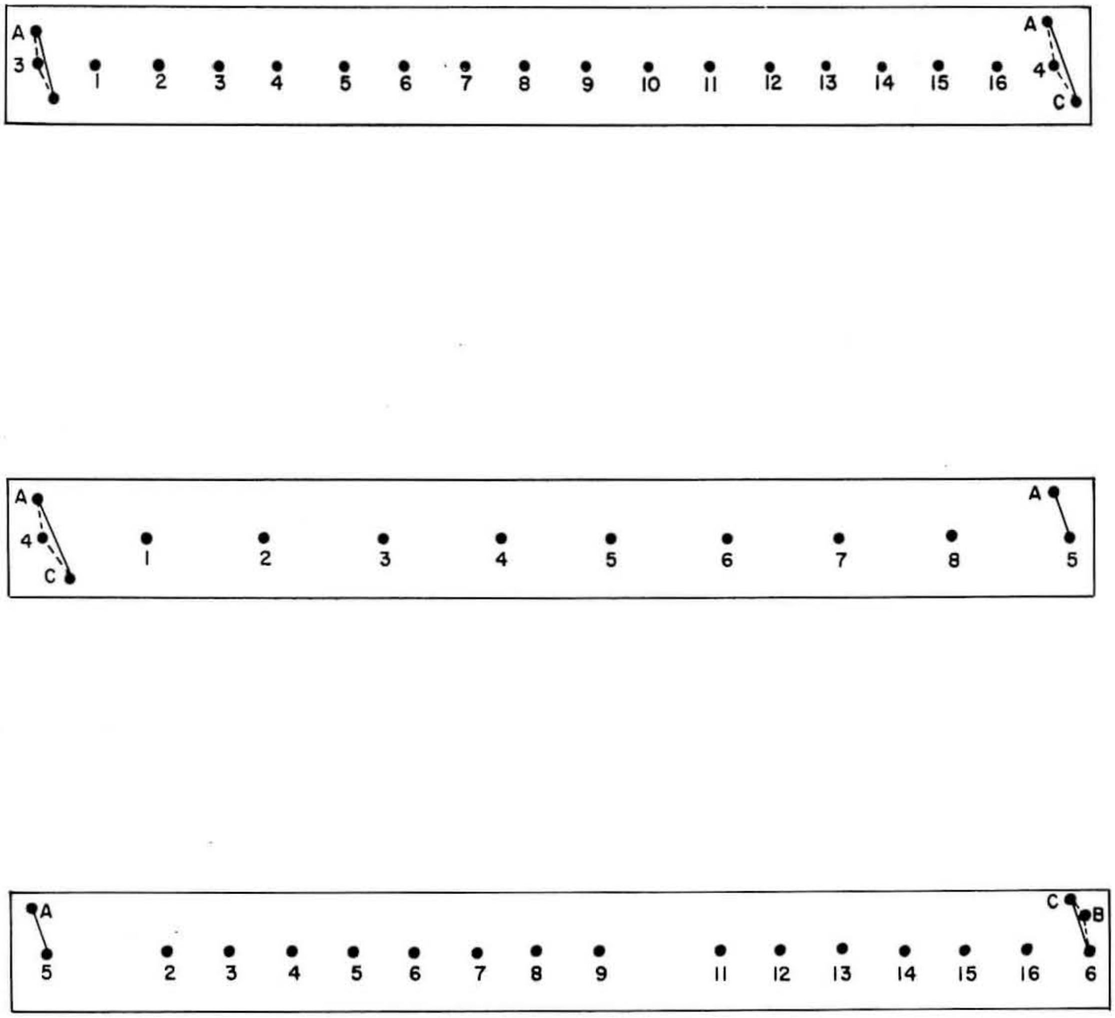

Fig. 6. Schematic arrangement of the 3 triangulated strips

TABLE I

Average Machine Coordinates

Leg	Point	x_{mm}	y_{mm}	H_m
4	3	11 000.00	1 000.00	+0.7
	A	10 941.81	1 167.47	-4.7
	C	11 007.52	832.15	-1.5
	3-1	10 608.37	981.05	-11.1
	3-2	10 210.99	961.31	-2.6
	3-3	9 903.34	945.97	-5.9
	3-4	9 438.27	922.50	-11.7
	3-5	9 065.72	903.48	+1.6
	3-6	8 679.76	883.61	-11.6
	3-7	8 252.08	861.32	-2.2
	3-8	7 910.62	843.41	-19.7
	3-9	7 479.35	820.47	-28.1
	3-10	7 102.44	800.12	-52.9
	3-11	6 673.79	776.76	+80.8
	3-12	6 299.22	756.14	+96.3
	3-13	5 938.93	736.25	+118.5
	3-14	5 521.73	713.14	+164.6
	3-15	5 154.37	692.57	+196.9
	3-16	4 805.16	672.97	+228.4
	A	4 461.63	812.22	+281.0
	4	4 469.91	653.97	+259.2
	C	4 477.59	441.45	+229.7

TABLE I
(continued)

Leg	Point	x_{mm}	y_{mm}	H_m
5	4	11 000.00	1 000.00	0.0
	A	10 986.42	1 157.81	+7.9
	C	11 019.78	788.02	-10.5
	4-1	10 324.96	938.47	+15.2
	4-2	9 646.51	875.99	+34.4
	4-3	8 976.16	813.78	+8.8
	4-4	8 301.36	750.55	+53.7
	4-5	7 626.77	687.25	+70.3
	4-6	6 957.34	624.55	+103.8
	4-7	6 287.34	561.69	+135.0
	4-8	5 617.28	498.61	+163.6
	5	5 272.29	466.07	+182.4
	A	5 259.45	638.47	+200.0
6	A	11 005.43	1 172.55	+6.3
	5	11 000.00	1 000.00	0.0
	5-2	10 350.82	1 007.77	+22.0
	5-3	10 020.24	1 011.62	+39.1
	5-4	9 689.25	1 015.46	+74.4
	5-5	9 351.75	1 019.49	+107.0
	5-6	9 024.95	1 023.65	+133.3
	5-7	8 695.59	1 027.92	+158.2
	5-8	8 352.38	1 032.47	+198.2
	5-9	8 018.55	1 036.93	+229.4
	5-11	7 352.49	1 045.95	+297.9
	5-12	7 021.13	1 050.45	+357.2
	5-13	6 688.16	1 055.05	+382.7
	5-14	6 358.27	1 059.77	+437.7
	5-15	6 031.98	1 064.61	+517.7
	5-16	5 704.75	1 069.49	+576.2
	6	5 372.84	1 074.25	+610.9
	B	5 380.61	1 201.74	+642.3
	C	5 387.10	1 293.50	+647.0

VII. Planimetric coordinate adjustment

A) Theory

Two known distances and their corresponding azimuths at both ends of the strip are all the data needed for adjustment purposes. The distances are approximately perpendicular to the base line, and their corresponding azimuths are in the same plane grid system.

The planimetric correction equations for longitudinal sections (y = constant) have the form:

$$\Delta x = a_o + a_1 x + a_2 x^2 + \dots.$$

$$\Delta y = c_o + c_1 x + c_2 x^2 + \dots.$$

More terms are neglected in this particular case. Since we can determine the origin of the grid syste, in any arbitrary position, a_o and c_o can be made zero and the equations become somewhat simplified:

$$\Delta x = a_1 x + a_2 x^2$$

$$\Delta y = c_1 x + c_2 x^2$$

Differentiating the above equations with respect to x, the scale correction becomes:

$$\frac{d\Delta x}{dx} = a_1 + 2a_2 x = \delta_s$$

And the azimuth correction:

$$\frac{d\Delta y}{dx} = c_1 + 2c_2 x = \delta\alpha$$

If x equals zero (and this is the case in the first model) then a_1 and c_1 are the scale and azimuth corrections, respectively, in the first model. Likewise in the last model a_2 and c_2 can be determined in a similar way. Having determined the coefficients, the correction equations are:

$$\Delta x = a_1(x-x_o) + a_2(x-x_o)^2 + c_1(y-y_o) + 2c_2(x-x_o)(y-y_o)$$

$$\Delta y = -c_1(x-x_o) - c_2(x-x_o)^2 + a_1(y-y_o) + 2a_2(x-x_o)(y-y_o)$$

For further details of the method, the reader is referred to Dr. Brandenberger's [2,4] and Dr. Ghosh's [6] publications.

B) Working equations

Measured base in first model = d_A

Given base in first model = D_A

Measured base in last model = d_E

Given base in last model = D_E

Initial x coordinate at first model = x_o

Last x coordinate at end of strip = x_E

Scale factor correction (first model) = $\delta s_A = \dfrac{D_A - d_A}{d_A}$

Scale factor correction (last model) = $\delta s_E = \dfrac{D_E - d_E}{d_E}$

Measured azimuth in first model = α_A

Given azimuth in first model = A_A

Measured azimuth in last model = α_E

Given azimuth in last model = A_E

$\delta\alpha_A = (A_A - \alpha_A)$; $\delta\alpha_E = (A_E - \alpha_E)$

Azimuth factor correction = $\delta\alpha = \delta\alpha_E - \delta\alpha_A$ expressed in radians

$a_1 = \delta s_A$

$$a_2 = \frac{\delta s_E - a_1}{2(x_E - x_o)}$$

$$c_1 = \delta\alpha_A$$

$$c_2 = \frac{\delta\alpha}{2\,(X_E - X_o)}$$

Since a_o, c_o and c_1 can be made zero (this will be done later during the transformation process), the equations become:

$$\Delta x = a_1(x-x_o) + a_2(x-x_o)^2 + 2c_2(x-x_o)(y-y_o)$$

$$\Delta y = -c_2(x-x_o)^2 + a_1(y-y_o) + 2a_2(x-x_o)(y-y_o)$$

The negative sign for c_2 is due to the fact that a positive azimuth correction results in a negative "y" correction.

Note: If more diatances and their corresponding azimuths are given, the scale and azimuth factors will be computed for each of them and the weighted mean will be taken. The weight is proportional to the distance.

The coefficients of legs 4, 5 and 6 are shown in Table II.

TABLE II

Leg No.4

Coefficients for Planimetric Adjustment

	Line	D_A(m)	d_A(m)	$SF_A (= \frac{DA}{dA})$	Weight	A_A	α_A	$\delta\alpha_A$
First Model	A-3	1 676.029	1 676.701	0.999598954	1	-0°43'58"	2°47'59"	3°31'57"
	3-C	1 679.279	1 680.184	0.999461612	1	-0°57'14"	2°33'54"	3°31'08"
	A-C	3 355.302	3 356.878	0.999530341	2	-0°50'37"	2°40'56"	3°31'33"
		D_E	d_E	$SF_E = (\frac{DE}{dE})$	Weight	A_E	α_E	$\delta\alpha_E$
Last Model	A-4	1 582.911	1 584.665	0.998893545	16	-2°41'13"	2°59'41"	5°40'54"
	4-C	2 124.926	2 126.587	0.999218795	21	-3°36'21"	2°04'10"	5°40'31"
	A-C	3 707.720	3 711.133	0.999080361	34	-3°12'49"	2°27'53"	5°40'42"

Weighted mean of δF_A = 0.999530312

δS_A = -0.000469688

Weighted mean of δF_E = 0.999079253

δS_E = -0.000920747

Weighted mean of $\delta\alpha_A$ = 3°31'32"

Weighted mean of $\delta\alpha_E$ = 5°40'41"

$d\alpha$ = -2°09'08"

$$a_1 = -\ 469\ 688 \times 10^{-9}$$
$$a_2 = +3\ 453\ 744\ 257 \times 10^{-18}$$
$$2a_2 = +6\ 907\ 488\ 514 \times 10^{-18}$$
$$c_2 = -2\ 876\ 331\ 284 \times 10^{-16}$$
$$2c_2 = -5\ 752\ 662\ 568 \times 10^{-16}$$

TABLE II

Leg No.5

Coefficients for Planimetric Adjustment

	Line	D_A	d_A	SF_A	Weight	A_A	α_A	$\delta\alpha_A$
First Model	A-4	1 582.911	1 583.932	0.999355401	16	$-2^\circ 41'13''$	$4^\circ 55'06''$	$+7^\circ 36'19''$
	4-C	2 124.926	2 124.946	0.999990587	21	$-3^\circ 36'21''$	$3^\circ 59'19''$	$7^\circ 35'40''$
	A-C	3 707.720	3 708.759	0.999719852	37	$-3^\circ 12'49''$	$4^\circ 23'08''$	$7^\circ 35'57''$
	Line	D_E	d_E	SF_E	Weight	A_E	α_E	$\delta\alpha_E$
Last Model	A-5	1 730.030	1 728.775	1.000725947		$-0^\circ 55'15''$	$4^\circ 15'33''$	$5^\circ 10'48''$

Weighted mean of SF_A = 0.999717882	Weighted mean of $\delta\alpha_A = 7^\circ 35'57''$
$\delta s_A = -0.000282118$	$\delta\alpha_E = 5^\circ 10'48''$
$\delta s_E = +0.000725947$	$\delta\alpha = 2^\circ 25'09''$

$$
\begin{aligned}
a_1 &= - \quad 282\ 118 \times 10^{-9} \\
a_2 &= - \quad 8\ 790\ 043\ 120 \times 10^{-18} \\
2a_2 &= - \quad 17\ 580\ 862\ 240 \times 10^{-18} \\
c_2 &= + \quad 3\ 681\ 825\ 750 \times 10^{-16} \\
2c_2 &= + \quad 7\ 363\ 651\ 500 \times 10^{-16}
\end{aligned}
$$

TABLE II

Leg No.6

Coefficients for Planimetric Adjustment

Line	D_A	d_A	SF_A	Weight	A_A	α_A	$\delta\alpha_A$
A-S	1 730.030	1 726.354	1.002285742		$-0^\circ 55' 15''$	$-1^\circ 48' 12''$	$-0^\circ 52' 57''$
Line	D_E	d_e	SF_E		A_E	α_E	$\delta\alpha_e$
B-6	1 282.740	1 277.266	1.004285716	13	$-3^\circ 40' 35''$	$-3^\circ 29' 16''$	$0^\circ 11' 19''$
C-6	2 205.145	2 197.132	1.003647027	22	$-3^\circ 54' 52''$	$-3^\circ 43' 17''$	$0^\circ 11' 35''$
C-B	922.432	919.892	1.002761193	9	$-4^\circ 14' 47''$	$-4^\circ 02' 45''$	$0^\circ 12' 02''$

$\delta s_A = + 0.002285742$

Weighted mean of $SF_E = 1.00365453725$

$\delta S_E = 0.00365453725$

$\delta\alpha_A = 0^\circ 52' 57''$

Weighted mean $\delta\alpha_E = 0^\circ 11' 36''$

$\delta\alpha = -1^\circ 04' 33''$

$$a_1 = + \quad 2\,285\,742 \times 10^{-9}$$
$$a_2 = - 1\,216\,384\,682 \times 10^{-17}$$
$$2a_2 = - 2\,432\,769\,364 \times 10^{-17}$$
$$C_2 = - 1\,668\,677\,541 \times 10^{-16}$$
$$2C_2 = - 3\,337\,355\,082 \times 10^{-16}$$

VIII. Elevation adjustment

The adjustment of the elevation can be divided into two parts:

A) Torsion or lateral correction, as function of lateral (ω) tilt.

B) Longitudinal correction as function of longitudinal (φ) tilt.

The first is usually a small correction in comparison to the second one and is due to systematic and random errors during the triangulation process.

The longitudinal correction includes also the earth curvature effect which propogates considerably in long strips regardless of instrumental and random errors.

In the present strip triangulations the difference between the two corrections is more evident since the first model of each leg was leveled along the "y" axis (ω) but not along the "x" axis (φ).

1) Torsion correction

The general torsion correction equation is

$$\Delta H_t = i_o y + i_1 xy$$

Differentiating the equation with respect to y yields

$$\frac{d\Delta H}{dy} t = i_o + i_1 x = \delta\omega$$

When x equals zero ; $i_o = \delta\omega$

When x equals x_E ; $i_1 = \dfrac{\delta\omega_E - i_o}{x_E}$

Since the torsion correction is a function of the "y" distance from the strip (x) axis (at y = 0 no torsion exists), the strip axis was approximately located with respect to the markers and the "y" distances were determined from this axis.

2) Longitudinal correction

This correction is expressed bu the parabolic equation

$$\Delta H = d_o + d_1 x + d_2 x^2$$

In order to determine the three coefficients, three known elevations must be given. Since barometric heights were read at each marker, three such elevations, taken at the beginning, middle and end of each leg, respectively, were used to determine the unknowns. However, since the barometric leveling is not a very precise one, a correction computed by the least squares method using all barometric observations was applied to the coefficients.

Combining the two corrections, the equation becomes

$$\Delta H = d_o + d_1(x-x_o) + d_2(x-x_o)^2 + i_o(y-y_o) + i_1(x-x_o)(y-y_o)$$

where the first three terms represent the longitudinal correction and the last two terms, the lateral correction.

Table III represents the coefficients computed for the elevation adjustment.

TABLE III

Coefficients for Elevation Adjustment

	Leg 4	Leg 5	Leg 6
i_o	$-745823389 \times 10^{-12}$	0	$+581395348 \times 10^{-13}$
i_1	$-1564602112 \times 10^{-16}$	$+1135617003 \times 10^{-16}$	$-502670026 \times 10^{-16}$
d_o	+2.021	-5.72	-19.46
d_1	$+498193221 \times 10^{-11}$	$+2259096654 \times 10^{-12}$	$+1072888888 \times 10^{-13}$
d_2	$-1065826027 \times 10^{-16}$	$-630838028 \times 10^{-16}$	$-1309205754 \times 10^{-16}$

Using the coefficients of Tables II and III, the adjusted coordinates were computed. Table IV contains the adjusted planimetric coordinates before the transformation. Since the adjusted elevations are final (i.e., the elevations will not change after the transformation), and in order to avoid unnecessary repitition, they are recorded in Table V rather than in Table IV.

TABLE IV

Adjusted Coordinates before Transformation

		x	y
Leg 4	C	110 075.25	8 322.20
	3	110 000.00	10 000.00
	A	109 918.21	11 673.99
	3-1	106 084.85	9 815.00
	3-2	102 108.01	9 631.20
	3-3	99 032.22	9 494.50
	3-4	94 383.81	9 295.66
	3-5	90 657.17	9 142.12
	3-6	86 795.77	8 991.86
	3-7	82 516.25	8 831.57
	3-8	79 098.92	8 710.04
	3-9	74 782.21	8 562.76
	3-10	71 008.59	8 440.19
	3-11	66 716.28	8 208.36
	3-12	62 964.84	8 199.74
	3-13	59 355.85	8 102.34
	3-14	55 176.03	7 998.22
	3-15	51 494.82	7 922.45
	3-16	47 994.92	7 837.75
	C	44 591.37	5 646.37
	4	44 599.68	7 770.99
	A	44 571.41	9 354.03

TABLE IV

(Continued)

Adjusted Coordinates before Transformation

		x	y
Leg 5	A	109 864.08	1 728.17
	4	110 000.00	1 720.47
	C	110 147.53	1 709.57
	4-1	103 253.93	1 747.97
	4-2	96 480.00	1 773.57
	4-3	89 793.14	1 743.47
	4-4	83 068.21	1 789.57
	4-5	76 351.79	1 796.79
	4-6	69 692.70	1 814.69
	4-7	63 035.26	1 824.70
	4-8	56 381.78	1 826.96
	A_s	52 757.66	1 826.07
	5	52 958.83	1 829.67

TABLE IV

(Continued)

Adjusted Coordinates before Transformation

		x	y
Leg 6	5	110 000.00	10 000.00
	A	110 054.39	11 729.44
	5-2	103 493.01	10 084.92
	5-3	100 179.22	10 132.51
	5-4	96 861.13	10 183.67
	5-5	93 477.36	10 240.76
	5-6	90 201.17	10 302.24
	5-7	86 899.02	10 368.60
	5-8	83 457.62	10 442.62
	5-9	80 110.22	10 518.74
	5-11	73 430.92	10 683.06
	5-12	70 107.80	10 770.32
	5-13	66 768.35	10 862.59
	5-14	63 459.65	10 959.26
	5-15	60 186.93	11 060.21
	5-16	56 904.54	11 165.28
	6	53 575.20	11 273.60
	B	53 677.08	12 551.80
	C	53 759.26	13 471.43

IX. Transformation

The transformation from one system to another requires at least two points in the first model, but more points are recommended if they are available.

The transformation procedure is based upon the centroid of all available control points in a model and the general formulas are

$$X = ax + by + c_1$$

$$Y = -bx + ay + c_2 \quad : \text{where}$$

$$Y_g = \frac{[Y]}{n} \qquad y_g = \frac{[y]}{n}$$

$$X_g = \frac{[X]}{n} \qquad x_g = \frac{[x]}{n}$$

$$Y_i^1 = Y_i - Y_g \qquad y_i^1 = y_i - y_g$$

$$X_i^1 = X_i - X_g \qquad x_i^1 = x_i - x_g$$

$$a = \frac{\Sigma[x_i^1 X_i^1 + y_i^1 Y_i^1]}{\Sigma[x_i^1 X_i^1 + y_i^1 Y_i^1]}$$

$$b = \frac{\Sigma[y_i^1 X_i^1 - x_i Y_i]}{\Sigma[x_i^1 X_i^1 + y_i^1 Y_i^1]}$$

$$c_1 = X_g - a \cdot x_g - b \cdot y_g$$

$$c_2 = Y_g - b \circ x_g - a \cdot y_g$$

The parameters for transferring Leg 5 to Leg 6 coordinate system are

$a = 0.090212969$

$b = 0.147464643$

$c_1 = 57104.186$

$c_2 = 14400.123$

The parameters for transferring Leg 4 into the new coordinate system of 5 are

$a = 0.997181616$

$b == 0.076781652$

$c_1 = 122326.499$

$c_2 = 3746.305$

By using the above transformation formulas and the corresponding parameters, Legs 4, 5 and 6 were placed in one coordinate system. Thus, Table V represents the final adjusted coordinates.

TABLE V

Final Adjusted Coordinates

Point	X_m	Y_m	H_m
C_6	53 759.26	13 471.43	2 058.76
B_6	53 677.08	12 551.80	2 056.56
6	53 575.20	11 273.60	2 028.76
5-16	56 904.54	11 165.28	2 043.68
5-15	60 186.93	11 060.21	2 025.66
5-14	63 459.65	10 959.26	1 989.39
5-13	66 768.35	10 862.59	1 973.40
5-12	70 107.80	10 770.32	1 983.14
5-11	73 430.93	10 683.06	1 957.81
5-9	80 110.22	10 518.74	1 946.65
5-8	83 457.62	10 442.62	1 940.07
5-7	86 899.02	10 368.60	1 921.76
5-6	90 201.17	10 302.24	1 914.32
5-5	93 477.36	10 240.76	1 903.14
5-4	96 861.13	10 183.67	1 883.25
5-3	100 179.22	10 132.51	1 857.39
5-2	103 493.01	10 084.92	1 845.25
A_5	110 054.39	11 729.44	1 836.07
5	110 000.00	10 000.00	1 829.67
4-8	113 454.82	9 956.71	1 826.96

TABLE V
(continued)

Point	X_m	Y_m	H_m
4-7	120 166.49	9 847.36	1 824.70
4-6	126 876.87	9 702.49	1 814.69
4-5	133 583.72	9 522.75	1 796.79
4-4	140 343.39	9 308.46	1 789.57
4-3	147 106.42	9 058.97	1 748.47
4-2	153 825.25	8 771.20	1 773.57
4-1	160 625.73	8 440.59	1 747.97
C_4	167 225.69	5 953.05	1 709.57
4	167 392.26	8 071.14	1 720.47
A_4	167 490.45	9 651.81	1 728.17
3-16	170 787.95	7 876.84	1 708.41
3-15	174 284.49	7 692.57	1 714.91
3-14	177 961.14	7 485.48	1 691.14
3-13	182 137.17	7 268.37	1 687.61
3-12	185 743.47	7 088.39	1 686.10
3-11	189 485.00	6 808.94	1 688.13
3-10	193 783.01	6 710.55	1 676.48
3-9	197 555.41	6 543.03	1 662.71
3-8	201 871.26	6 358.45	1 663.24
3-7	205 288.29	6 217.25	1 649.13
3-6	209 568.06	6 048.50	1 642.06
3-5	213 430.11	5 901.85	1 653.02
3-4	217 158.03	5 768.82	1 639.24
3-3	221 808.61	5 610.19	1 634.70
3-2	224 886.23	5 510.34	1 628.89
3-1	228 865.97	5 388.27	1 605.57
A_3	232 831.27	6 947.78	1 542.84
3	232 784.29	5 272.14	1 599.84
C_3	232 730.50	3 593.19	1 598.54

The horizontal and vertical position of the markers are shown in in Figures 7 and 8, respectively.

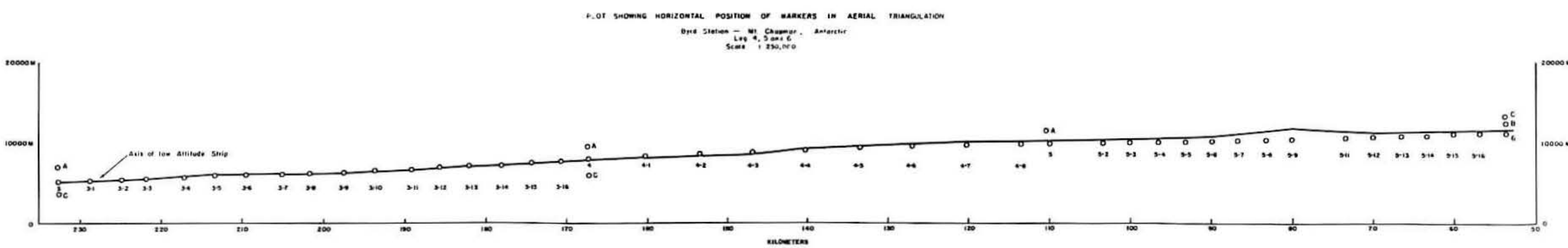

Figure 7

Horizontal position of the markers

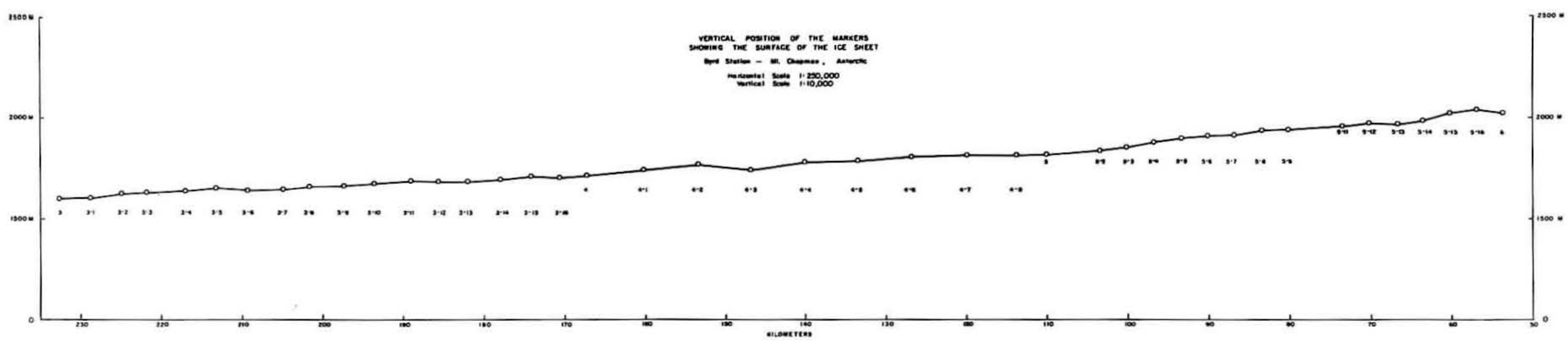

Figure 8

Vertical position of the markers

X. Connection of the strip to Mt. Chapman

The strip has to be connected to a "fixed" base which does not move with the ice sheet. Only by referring to this fixed base, it is possible to determine later the rate of movement of the targets.

During the field work process, several peaks of Mt. Chapman were intersected from the markers at the last base line of the strip (Leg 6) and their coordinates were computed in a local grid system. These coordinates were transformed to the newly established grid system, and are shown in Table VI_a .

The coordinates of the last base line as well as those of the mountain peaks were plotted in the scale 1:5 000 and five models were bridged between them. The first model, including the last base line, was absolutely oriented, based on the adjusted coordinates obtained previously. The location of the selected peaks were found by following the triangulation procedure with the coordinatograph connected to the instrument. Since the first model was absolutely oriented and since only five models were bridged, the accumulated error was very small, and the identification of the peaks in the fifth model was obtained by bringing the coordinatograph pencil to the previously plotted targets on the plot sheet and observing in the instrument the corresponding locations. A very small correction in scale and leveling was applied to the last model, to read the given coordinates.

The selected peaks were not in all cases the highest spots on the mountain and some of them were not visible because of the deep shadows, therefore, some more control points were selected during the operation and the machine coordinates were transformed into ground coordinates in the grid system previously established (Table VI_b).

These points, together with the field points, will provide data for orienting the first model (including the mountain) for future work.

In order to locate and identify the selected control points in the future, a map of the top of the mountain was plotted (Figure No. 12). Also, a detailed description of each point in a larger scale was plotted to facilitate the identification. The map will also enable to determine whether topographical changes have occured on the mountain.

TABLE VI_a

Transformation of Mountain Points into the Grid System

$$\left.\begin{aligned} a &= 1.000020782 \\ b &= 0.015235827 \\ c_1 &= 54099.70 \\ c_2 &= 11265.42 \end{aligned}\right\} \text{Transformation Parameters}$$

	Field Coordinates		Transformed Coordinates to Common System and Ground Elevations		
	X'_m	Y'_m	X_m	Y_m	H_m
A_1	7 629.2	1 307.7	46 490.26	12 689.39	2 573.76
A_2	7 610.0	1 250.0	46 508.58	12 631.38	2 605.96
B_1	6 180.0	620.0	47 929.02	11 979.59	2 732.46
C_1	6 286.0	583.0	47 822.45	11 944.20	2 738.16
E_1	6 320.0	10.0	47 779.72	11 371.71	2 236.16
E_2	6 110.0	-40.0	47 988.96	11 318.51	2 234.06
F_1	6 220.0	-190.0	47 876.68	11 170.19	2 334.76
G_1	5 752.2	-445.5	48 340.59	10 907.55	2 428.56
M_1	5 889.6	-1 075.1	48 193.60	10 280.03	2 116.16
M_2	5 897.5	-1 074.5	48 185.71	10 280.71	2 121.96

TABLE VI_b

Transformation of New Mountain Points into the Grid System

$$\left.\begin{array}{l} a = 10.00330454 \\ b = 0.19694606 \\ c_1 = 43286.89 \\ c_2 = 1480.40 \end{array}\right\} \text{Transformation Parameters}$$

	Machine Coordinates		Transformed Coordinates to Common System and Ground Elevations		
	x_{mm}	y_{mm}	X_m	Y_m	H_m
G_1	486.40	951.98	48 340.59	10 907.55	2 428.6
C_1	432.75	1 054.59	47 822.45	11 944.20	2 738.2
A_1	298.02	1 126.37	46 490.26	12 689.39	2 573.8
K	472.09	891.75	48 184.98	10 307.87	2 145.7
S	340.35	1 114.30	46 910.97	12 560.05	2 478.0
I	380.48	1 015.02	47 292.85	11 559.02	2 558.0
M	348.70	996.90	46 971.37	11 384.02	2 544.2
H	359.29	961.30	47 070.30	11 025.82	2 221.7
A	444.45	976.17	47 925.11	11 157.79	2 331.6

XI. Analysis of the obtained accuracy

It is impossible, in this case, to determine directly the accuracy obtained in the process of aerial triangulation since there are no ground control points in the strip which were not used during the process and would be available for an accuracy analysis. Nevertheless, an attempt is made to examine several stages of the process and estimate roughly the accuracy obtained.

A) Relative orientation accuracy

It was pointed out previously, that because of the undetailed background, it was rather difficult to observe remaining Y-parallaxes in the model. However, the remaining Y-parallax never exceeded the size of the measuring mark (0.04 mm), yet the required accuracy of a 1/4 of that size could not always be attained.

B) Pointing accuracy

By repeated measurements it was found that the pointing accuracy ranges from ±0.01 mm to ±0.05 mm (0.5 m on ground) in planimetry, and ±0.1 m to ±1.0 m in elevation, depending on the location and nature of the point observed. On the markers the pointing accuracy, in general, is far better than the pointing accuracy on the ~~crevasses.~~ sastrugi

C) Model connection accuracy

By comparing the corresponding readings of each point in Model N and model N+1 it is possible to estimate the standard error of one model connection, which affects the overall accuracy of the aerial triangulation performed.

The center transfer points were always set to read the same coordinates as in the previous model; therefore only the wing points were used for this analysis. Moreover, since the average coordinates were eventually used, the residuals were taken from respective averages. The formula used for evaluating the standard errors is:

$$M = \pm \sqrt{\frac{[vv]}{n}}$$

where v is the residual and n is the number of points used. These standard errors are shown in Table VII.

TABLE VII

Standard Errors of Model Connection

Leg No.	At Instrument mm		On Ground m
4	$M_X = 0.01$	=	0.1
	$M_Y = 0.02$	=	0.2
	$M_H =$	=	1.6
5	$M_X = 0.007$	=	0.07
	$M_Y = 0.014$	=	0.14
	$M_H =$	=	1.4
6	$M_X = 0.01$	=	0.1
	$M_Y = 0.017$	=	0.17
	$M_H =$	=	1.2
Or generally	M position	=	0.13
	M'elevation	=	1.4

However, one must keep in mind that the standard errors of the mid-points (which were not included in this computation) are far better than those obtained from the wing points because along the strip axis the resolution is highest, the distortion is lowest and the torsion has no effect. Since the markers were located along this axis the respective planimetric accuracy, and especially the elevation accuracy, is much better.

D) Planimetric accuracy

Table $VIII_a$ and $VIII_b$ represent the closing errors at the end of each leg. However, the closing error is not a sufficient measure of accuracy and the small values for the closing errors are mainly a check of the computations and indicate that the procedure followed is well performed. In order to perceive the magnitude of the expected accuracy, several empirical formulas were developed. Yet all these formulas cannot be completely adapted to this case.

$$m_P = h \cdot \sqrt{N} \cdot 0.1 \text{ m}$$

where

m_P = standard residual position error = $\pm \sqrt{M_X^2 + M_Y^2}$

h = flight height in km

N = number of models

But this formula is limited to a strip of models having ground control points at the beginning, middle and end of the strip. Neither of these requirements exist in the present case. Substituting $h = 3$ km and $N = 45$ the above equation generates a result of $m_P = \pm 2$ m which seems to be too low considering the fact that control points are available only at the beginning and end of the strip, and that each strip consists of 45 models.

Dr. B. Hallert's formula is closer to the case at hand since it is adaptable by having control points at the beginning and end of the strip only.

The standard error expected in the middle of the strip where $p = \frac{n}{2}$

$$mx = \mu \frac{h}{c} \sqrt{p \frac{(n-p)}{3n} \left\{2p(n-p) + 1\right\}}$$

$$my = \mu \frac{h}{c} \sqrt{\frac{2p(n-p)}{9n} \left\{p(n-p) + 2\right\}}$$

where

μ = standard error of the parallax measurement = 0.01 mm

h = flight height in m

c = focal length

n = number of models

p = a photo in the strip

substituting

h = 3 000 m

c = 153 mm

μ = 0.01 mm

n = 45

$p = \frac{n}{2}$

$mx = \pm 12.33$ m

$my = \pm 7.12$ m

$m_P = \sqrt{m_x^2 + m_y^2} = .14.6$ m

This is the standard error in position expected in the middle of the strip. Since in this location we may expect the maximum error, the standard error along the whole strip is approximately 1/3 of this amount, thus the standard residual position error is

$m_P = \pm 5$ m .

Dr. Hallert did not indicate the reliability of this formula and generally it is doubtful whether theoretical formulas can be applied to practical tests of accuracy determination. Moreover, it is doubtful whether Hallert's formulas can be applied to independent geodetic control method since two bases are measured independently at the beginning and end of the strip and the errors between them must be added to the randon errors usually existing in aerial triangulation.

Actual tests are needed in order to develope an empirical formula for the accuracy of aerial triangulation using the method of independent geodetic control. Such empirical formulas do not exist yet.

However, in this case,the result obtained appears to be probable. Re-examining Dr. Brandenberger's formula, and the unfavorable conditions in the present aerial triangulation it is logical to assume approximately twice the standard position error in case the number of models involved is double the amount required for this formula. Doing so, the results obtained from the two formulas will agree to a certain extent.

TABLE $VIII_a$

Closing Errors after First Adjustment

Base Line	Given Value m	Adjusted Value m	Closing Error m
Leg 4			
A-3	1 676.029	1 675.591	-0.438
3-C	1 679.279	1 679.406	+0.127
A-C	3 355.302	3 355.306	+0.004
A-4	1 582.911	1 584.331	+1.420
4-C	2 124.926	2 126.132	+1.206
A-C	3 707.720	3 710.345	+2.625
Closing error in azimuth = 8"			
Leg 5			
A-4	1 582.911	1 583.492	+0.581
4-C	2 124.926	2 124.331	-0.595
A-C	3 707.720	3 707.701	-0.019
A-5	1 730.030	1 731.055	+1.025
Closing error in azimuth = 44"			
Leg 6			
A-5	1 730.030	1 730.292	+0.262
B-6	1 282.740	1 282.254	-0.486
C-6	2 205.145	2 205.524	+0.379
C-B	922.432	923.295	+0.863
Closing error in azimuth = 13"			

TABLE $VIII_b$

Closing Errors after Second Adjustment
(Improvement of Leg 4 and 5 only)

Leg 4

Corrected Coefficients
$$\begin{cases} a_1 = +\,0.000103478 \\ a_2 = +\,621625574 \times 10^{-17} \\ C_2 = 0 \end{cases}$$

Base Line	Given Value m	Adjusted Value m	Closing Error m
A-3	1 676,029	1 675.732	-0.297
3-C	1 679.279	1 679.476	+0.197
A-C	3 355.302	3 355.314	+0.012
A-4	1 582.911	1 583.211	+0.300
4-C	2 124.926	2 124.622	-0.304
A-C	3 707.720	3 707.715	-0.005

Closing error in azimuth = 1"

Leg 5.

Corrected Coefficients
$$\begin{cases} a_1 = 0.000093432 \\ a_2 = 826784534 \times 10^{-17} \\ C_2 = -242756966 \times 10^{-17} \end{cases}$$

Base Line	Given Value m	Adjusted Value m	Closing Error m
A-4	1 582.911	1 583.461	+0.550
4-C	2 124.926	2 124.362	-0.564
A-C	3 707.720	3 707.710	-0.010
A-5	1 730.030	1 730.450	+0.015

Closing error in azimuth = 0"

E) Elevation accuracy

In regard to the elevation accuracy the following quotation must be kept in mind. "Looking to the uncertainties inherent in barometric leveling as performed in taking photography, it is very difficult to set a value on the degree of accuracy which may be expected... with a single aneroid in the field... the error of any single observation may be anywhere between 10 and 100 ft." [Clark, Vol II, p 459, 4th Edition.]

It is clear that if the field observations are not determined with high accuracy, the accuracy of the photogrammetric observations based upon the field data dannot be high. Thus, the residuals between barometric and adjusted elevations would not generate a reliable value of the standard error of the barometric elevation. (Table IX and Figure 9)

TABLE IX

Residuals of Elevation

Leg 4 Point	Barometric Elevation m	Adjusted Elevation m	Residual m
3	1 623.00	1 625.02	- 2.02
3-1	1 625.00	1 630.75	- 5.75
3-2	1 649.00	1 654.07	- 5.07
3-3	1 657.00	1 659.88	- 2.88
3-4	1 666.00	1 664.44	+ 1.58
3-5	1 684.00	1 678.20	+ 5.80
3-6	1 671.00	1 667.24	+ 3.76
3-7	1 681.00	1 674.31	+ 6.69
3-8	1 698.00	1 688.42	+ 4.48
3-9	1 696.00	1 687.89	+ 8.11
3-10	1 707.00	1 701.66	+ 5.34
3-11	1 708.00	1 713.31	- 5.31
3-12	1 702.00	1 711.28	- 9.28
3-13	1 699.00	1 712.79	-13.79
3-14	1 715.00	1 716.32	- 1.32
3-15	1 723.00	1 740.09	-17.09
3-16	1 734.00	1 733.59	+ 0.41
4	1 741.00	1 745.65	- 4.65

TABLE IX
(Continued)

Leg 5 Point	Barometric Elevation m	Adjusted Elevation m	Residual m
4	1 745.65	1 739.93	+ 5.72
4-1	1 762.65	1 767.43	- 4.78
4-2	1 787.65	1 793.03	- 5.38
4-3	1 773.65	1 767.93	+ 5.72
4-4	1 814.65	1 809.03	+ 5.62
4-5	1 823.65	1 816.25	+ 7.40
4-6	1 826.65	1 834.15	- 8.50
4-7	1 841.65	1 844.16	- 2.49
4-8	1 842.65	1 896.92	- 3.77
5	1 845.65	1 849.13	- 3.48
Leg 6			
5	1 849.13	1 829.67	+19.66
5-2	1 853.13	1 845.25	+ 7.88
5-3	1 856.13	1 857.39	- 1.26
5-4	1 874.13	1 883.25	- 9.12
5-5	1 887.13	1 903.14	-16.01
5-6	1 902.13	1 914.32	-12.19
5-7	1 912.13	1 921.76	- 9.63
5-8	1 940.13	1 940.07	+ 0.06
5-9	1 954.13	1 946.65	+ 7.48
5-11	1 960.13	1 957.81	+ 2.32
5-12	1 990.13	1 983.14	+ 6.99
5-13	1 984.13	1 973.40	+10.73
5-14	2 000.13	1 989.39	+10.74
5-15	2 037.13	2 025.66	+11.47
5-16	2 050.13	2 043.68	+ 6.45
6	2 035.13	2 028.76	+ 6.37

Leg 4 MH = $\pm$ 7.56 m

Leg 5 MH = $\pm$ 5.85 m

Leg 6 MH = $\pm$ 9.25 m

or generally

MH = $\pm$ 7.5 m

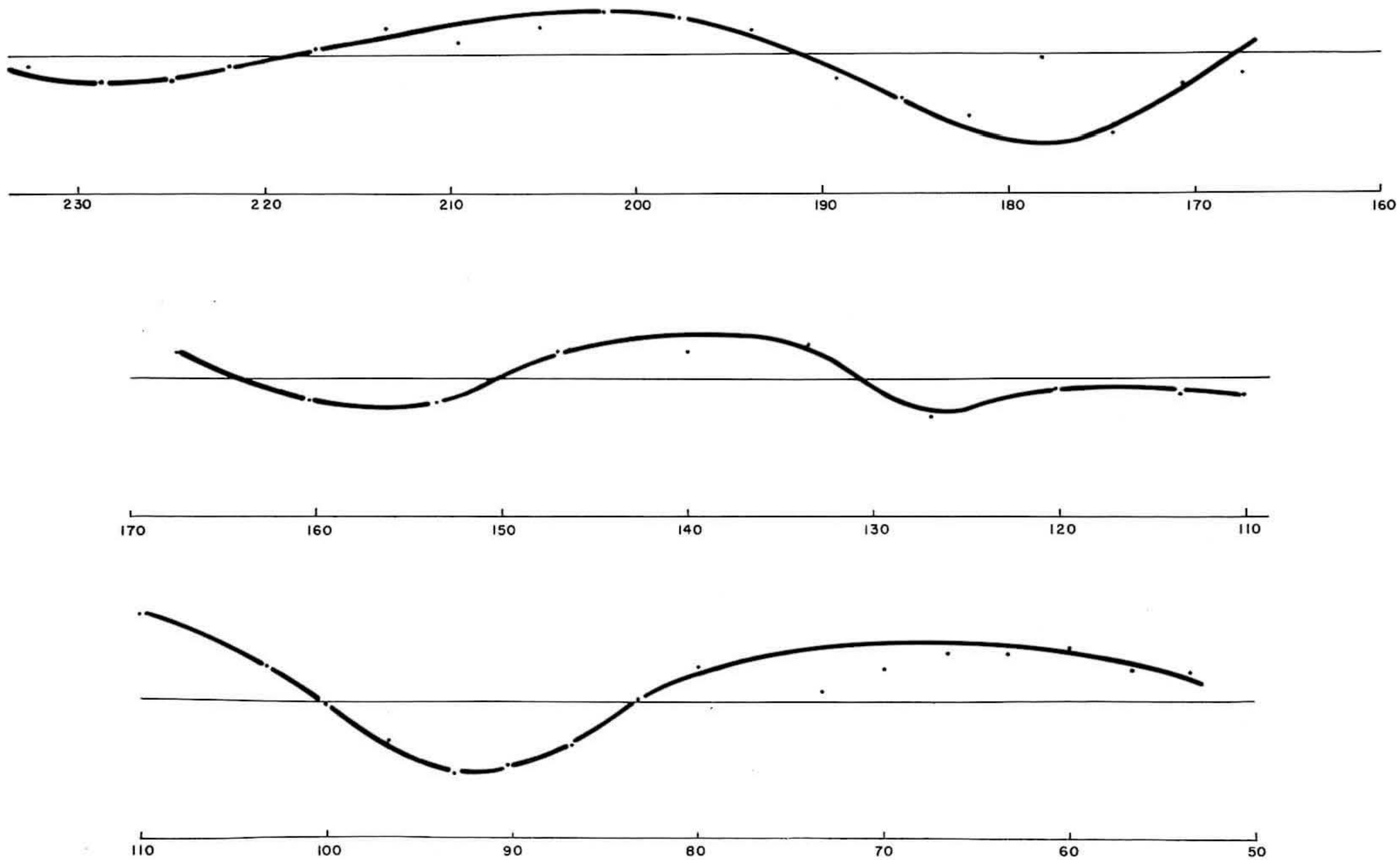

Fig. 9. Graph of residual elevation errors

Another attempt was made to analyze the elevation accuracy, using the instrumental "bz" element. This method is based upon the assumption that the airplane flew with minimum deviations from a constant altitude, and that all elevation errors (or the major part of them) would affect mainly the "bz" element. Since the "bz" of each model was recorded together with all other elements, these values were plotted as elevations versus the x-coordinate (model scale 1:10 000 i.e. 1 mm = 10 m). On the same graph the differences between the instrumental readings and the "known" (final adjusted elevations) values were plotted. (Figure 10.)

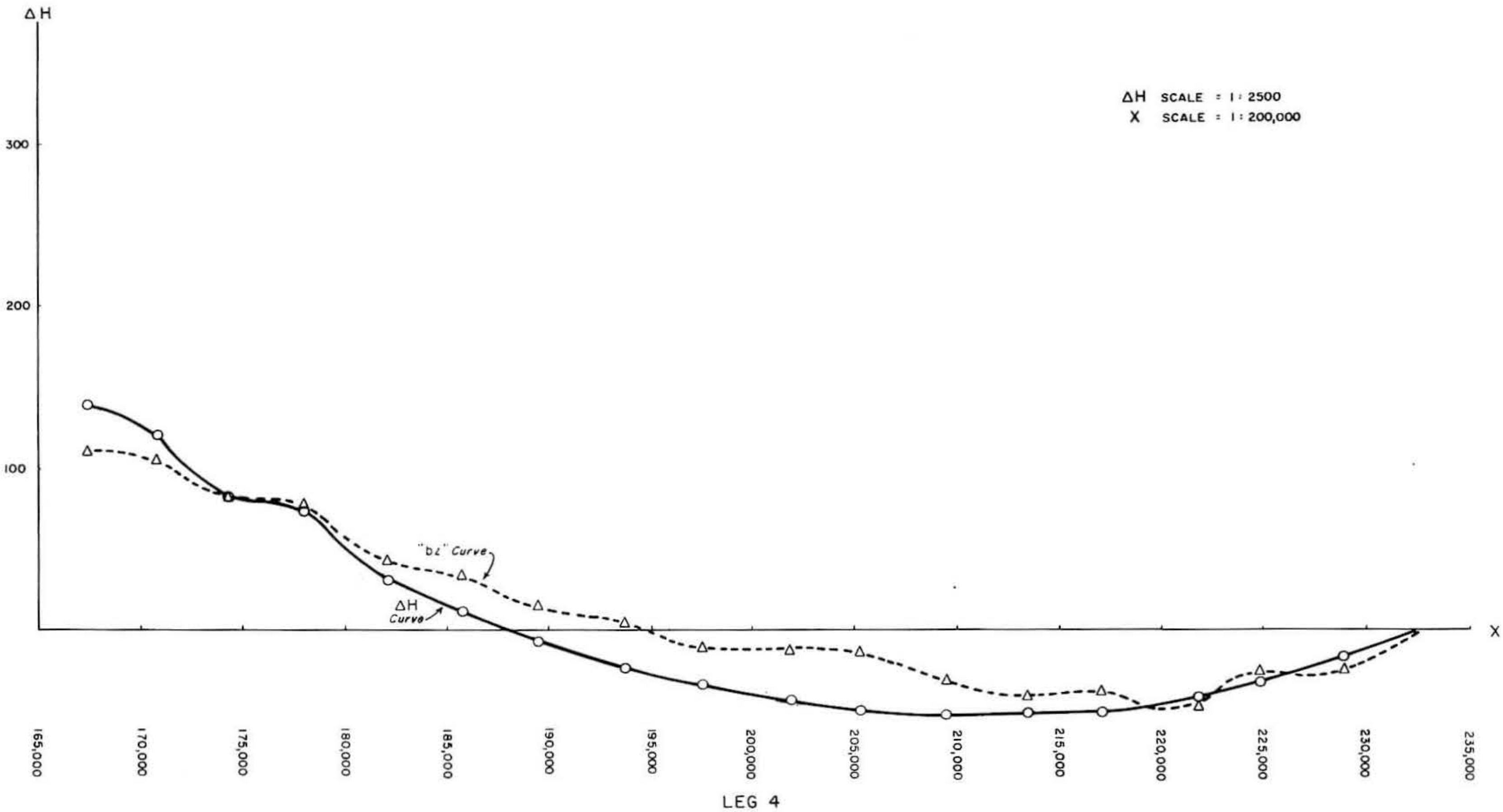

Fig. 10. Graph of "bz" curves-elevation differences curve

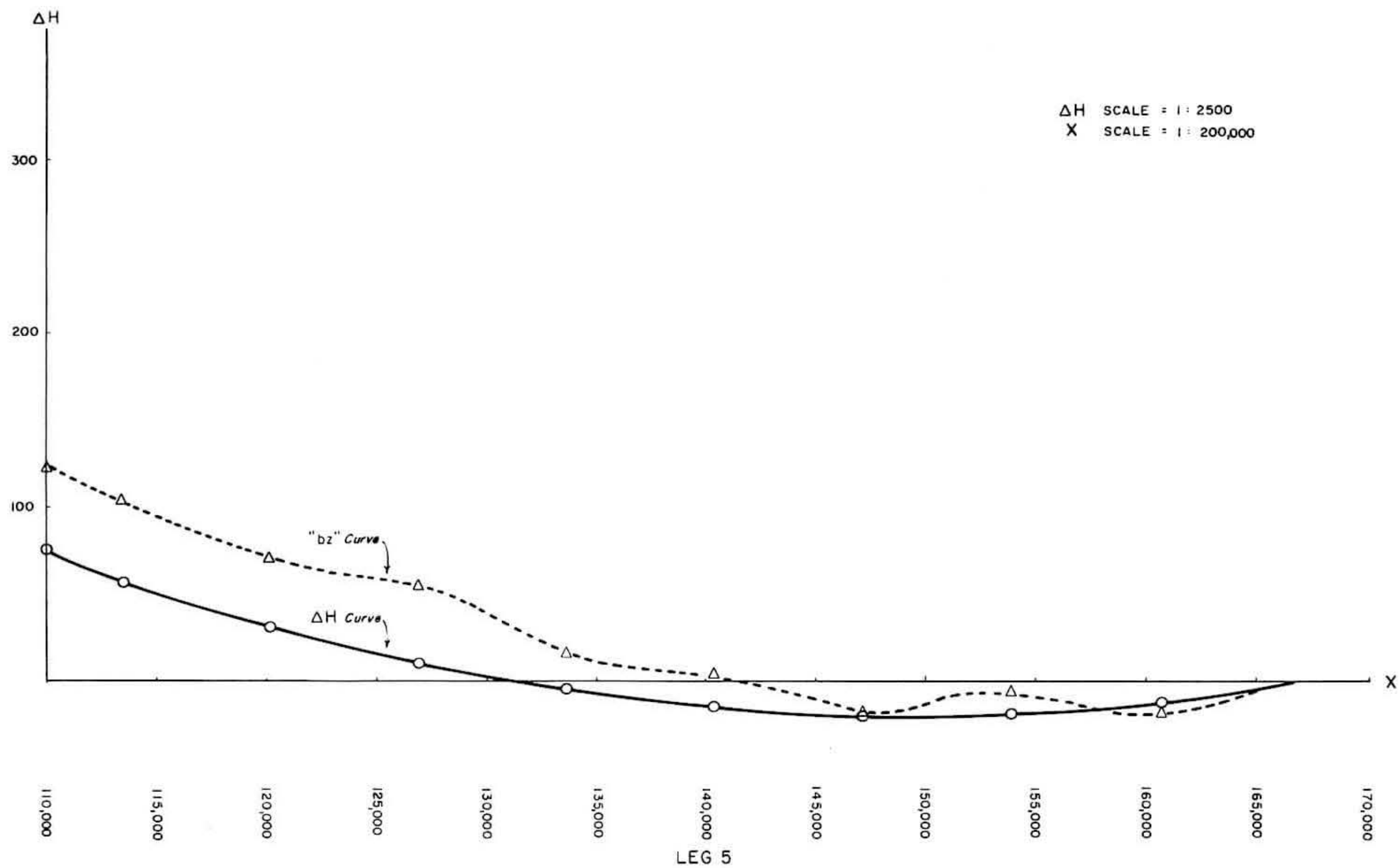

Fig. 10 (Continued)

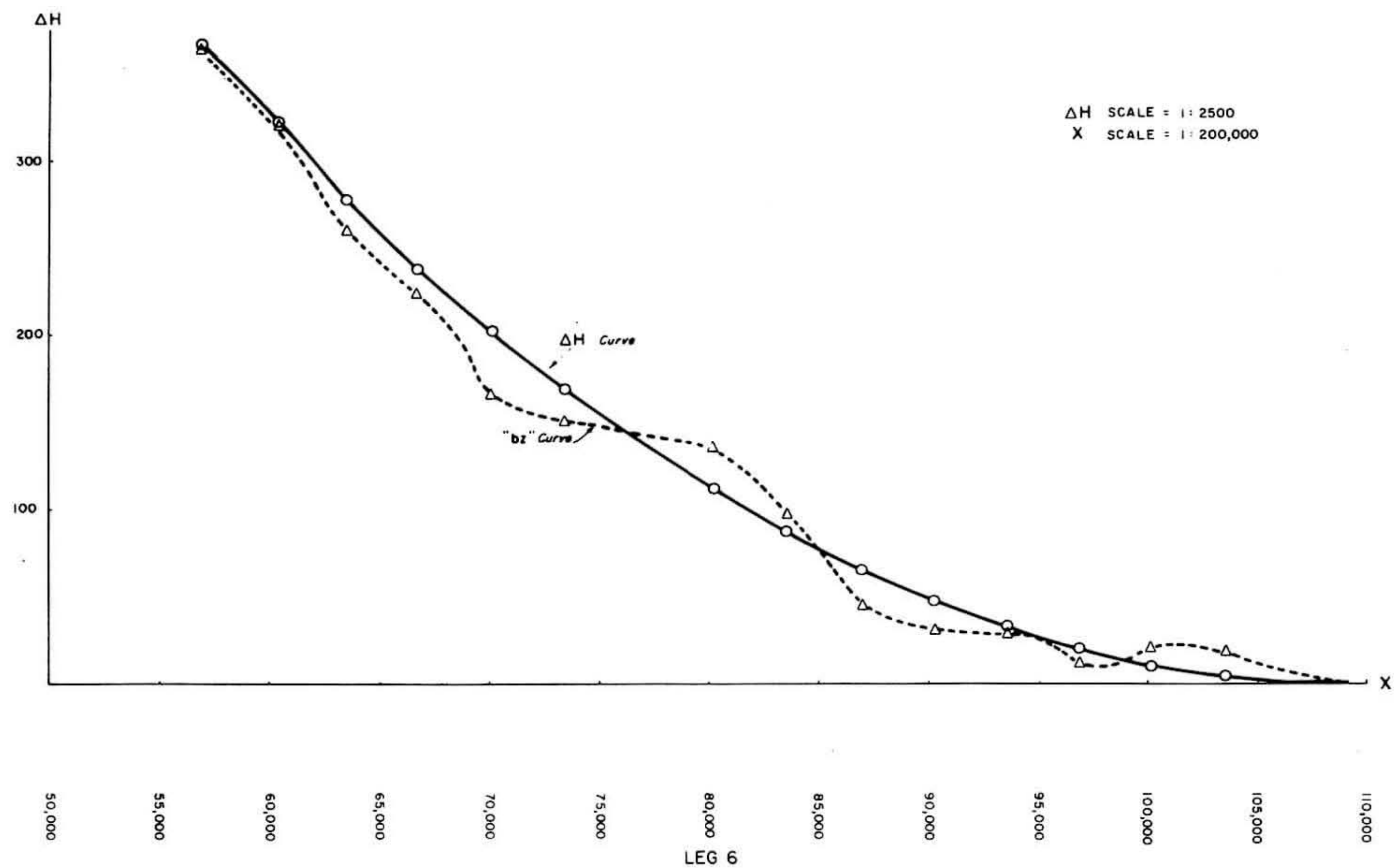

Fig. 10 (Continued)

Looking at the graph the following conclusions may be drawn:

1) The major part of the elevation errors are systematic errors. This fact agrees with the theory that in long strips systematic errors exceed random errors, and proves that the operation process was well done.

2) The error curve balances the "bz" curve in Legs 4 and 6 which proves that the adjusted elevations really forms the best fitted curve.

3) This experiment may serve as a basis for a new method of adjusting elevations, if no control points exist along the strip and if the required accuracy is not too high (for small scale mapping) i.e., applying the best fitted curve between the plotted "bz" values and reducing the curve to the reference datum.

To sum up the accuracy analysis, it can be stated that the absolute accuracy of the results obtained cannot be exactly determined for the reasons mentioned earlier. However, in this work, the determination of the absolute accuracy is not very essential. The emphasis should be placed upon the relative accuracy which, by using photogrammetric means, is generally higher than the absolute accuracy.

The results obtained can be regarded as errorless for comparison purposes in future work and the effect of the absolute accuracy upon the relative accuracy will not significantly affect the determination of the relative movement of the ice sheet.

XII. Discussion, conclusions and recommendations

In view of the comments made in this report and in view of the experience gained in this work, the following recommendations seem to eliminate most of the problems encountered previously and to provide better data in future work.

A) Reconnaissance

The targets should be examined to determine whether they are still on the ground, not covered by snow, and in good condition.

It is highly recommended to add one more target to each of the single targets. If this cannot be done because of transportation problems, at least 14 more targets need to be added and erected 1.5 km to each side of the principal targets along the strip axis. These targets are available by removing targets E and H (Forest's preliminary report [1], p 14) from their present locations.

B) Photo flight and photography

The camera to be used should be of the highest quality i.e., a small lens distortion and high resolving power wide-angle camera. The camera should be checked as to its overlap control.

The airplane carrying the camera should head from Byrd Station to Mt. Chapman without taking any photographs. During reconnaissance flight the exact azimuth of the flight should be determined by observing the principal targets on the ground. In addition it would be helpful to drop pieces of cloth or colored powder from the airplane and scattered along the length of the strip. (These identifying features will facilitate the establishment of stereoscopic vision later.)

Having established the correct heading, the airplane will then return and fly the strip again, this time taking the photographs. In order to cover a large area per photo and still be able to observe the ground markers, the airplane altitude should be 4,500 to 5,000 m above sea level. It was explained previously that two photo strips taken at different altitudes will not help very much, therefore only <u>one altitude</u> should be flown.

To insure proper coverage the overlap should be 80 percent. If this overlap is maintained along the entire strip, every second photograph will be used. In case of unpredicable irregularities the 80 percent overlap insures several possible combinations to be used in order to obtain the required overlap for aerial triangulation purposes. A reliable statoscope (with accuracy of ± 1 m) should be operated during the photography.

C) Field work

1) The theodolite is set at one end of the base line, the nearest to Mt. Chapman. Directions as well as vertical angles are taken to two clearly identified peaks of the mountain (I, II) and to a point (a) (see Figure 11) on the axial line.

2) The theodolite is to be transfered to the other end of the base line while its length is taped or measured with a tellurometer and directions plus vertical angles are measured to the same points as in Step 1.

3) The theodolite is set at (a) and the horizontal angle measured between the end point of the base and the next station. The stations between (a) and (b) are chosen as far as possible from each other and ~~must~~ do not have to be located at the markers. Rather, they will be chosen in such a way as to provide the maximum distance from which the measuring rod can still be sighted by the theodolite, thus reducing the number of stations between (a) and (b) as much as possible.

4) On reaching point (b) vertical and horizontal angles are taken to the two end points (U,L) of the base line.

5) From (L) points b, U and C are sighted horizontally and vertically.

6) Transfering the theodolite to (U) while taping the distance, and sighting (b), (L) and (C).

7) From (C) repeat the same procedure as was done from (a) and so on to the end of the strip at Byrd Station.

8) Barometric heights will be recorded only at stations a, b, c, etc. These heights will be read from three aneroids in order to obtain a reliable value, as suggested by Clark.

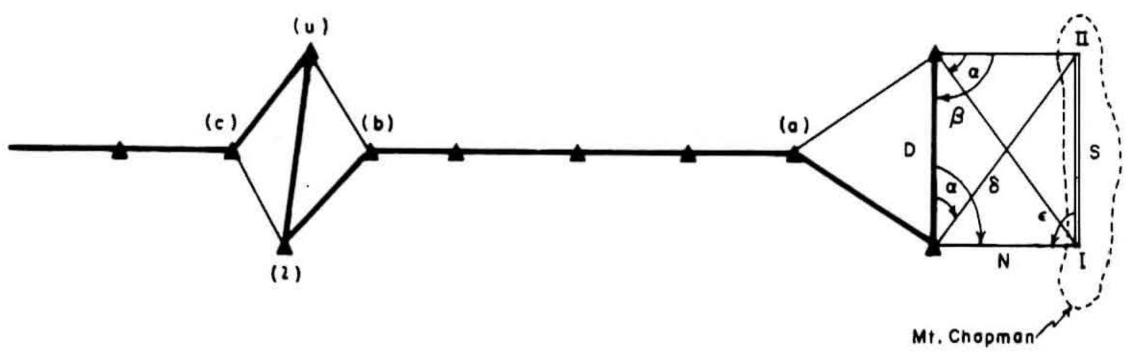

Figure 11

Scheme of field work

D) Computation of field data

Using the method of independent geodetic control we need to know:

1) The lengths of the base lines.

All base lines are taped or measured by a tellurometer thus the first requirement is fulfilled.

2) The azimuths of the base lines.

From Steps 1 and 2 of the field work the following data would be obtained:

Angles α, β, γ, δ

Base length D

Angle ϵ and distance S between the two peaks of the mountain can be computed by using Hansen's method.

Now, distance S is fixed and giving it any arbitrary azimuth, the azimuth of base line D can be computed according to the following formulas:

$$A_H = A_S + \epsilon - 180^{\circ}$$

$$A_D = A_N + \delta - 180^{\circ} = A_S + \epsilon + \delta - 360^{\circ}$$

In the same way the azimuth is carried out along the traverse to each of the base lines and thus, the second requirement is fulfilled.

Note: The same azimuth assigned to line S will be used always in future work therefore it can be regarded as a given quantity.

3) Sufficient data in the first and last models of each leg to enable absolute orientation of these models.

To establish the absolute orientation of a model, at least one distance and two elevation differences along the x and y axes are required. The set-up at each base line provides more than this minimum. The distance of the base line is measured, the distances to the other two targets can easily be determined trigonometrically. The elevation differences between the four points are obtained from trigonometric leveling. This procedure insures sufficient data also in case one of the targets should not appear in the model.

E) Instrumental operation

Having the statoscope data, the aero leveling method should be used for the aerial triangulation, rather than the aeropolygon method. This method will enable bridging all models without breaking the strip into legs.

Glass diapositives are recommended for use rather than negatives to insure definite size of the photo.

In addition it is recommended to use pricked points for transfer purposes rather than natural points such as crevasses. The final computations, and the adjustment of the measurements, will be done in the same way as this work was accomplished.

XIII. Summary

The first part of the project, dealing with the determination of the movement of the ice sheet in the Antarctic, is completed. The field work and the computations of the data were covered in the report of Dr. R. B. Forrest to the National Science Foundation. This report covered the triangulation process, gave the final coordinates of the markers placed in the area, and evaluated the work. The report also includes detailed recommendations for future work.

The future work for this project will include repetition of the field and instrumental observations. The desired information on the actual ice movement will be obtained only after these observations have been processed and analyzed. The present report gives the basic data which will be compared later with future data.

Not all the entire strip could be triangulated because of the gap in the middle, and because of other problems which have been discussed previously in this report.

It can be stated here that the work accomplished shows clearly the potential of photogrammetric methods in determining the rate of the ice movement.

It is strongly believed that if better equipment is used and if the field work is performed as suggested - results will be obtained with high accuracy.

XIV. References

[1] Forrest, R. B., "The Use of Photogrammetric Methods to Investigate Surface Movement of the Antarctic Ice Sheet", preliminary report to National Science Foundation, Contract G-23006, RF Project 1444, 1963.

[2] Brandenberger, A. J., "Strip Trianbulation with Independent Geodetic Controls; Triangulation of Strip Quadrangles", Publication of the Institute of Geodesy, Photogrammetry and Cartography, No. 9, The Ohio State University, 1959.

[3] Brandenberger, A. J., "The Practice of Spatial Aerial Triangulation", Photogrammetric Institute of the Federal Institute of Technology, Zurich, Switzerland, 1951.

[4] Brandenberger, A. J. and Laurila, S., "Aerial Triangulation by Least Squares", final report Contract No. DA-44-009 Eng 2638, Engineer Research and Development Laboratories, Fort Belvoir, Virginia, RF Project 647, The Ohio State University Research Foundation, 1956.

[5] Brandenberger, A. J., "Some Considerations About Error Propagation in Strip Triangulations-Attainable Accuracy", publication of the Institute of Geodesy, Photogrammetry, and Cartography, No. 8, The Ohio State University, 1958.

[6] Ghosh, S. K., "Strip Triangulation with Independent Geodetic Control", Photogrammetric Engineering, Vol 5, 1962. (Nov)
XXVIII No.

[7] Hallert, B., "A Theoretical Investigation of Aerial Triangulation as a Problem of Maxima and Minima", Photogrammetric Engineering, Vol 5, 1958.

Project
Supervisor A. J. Brandenberger Date Oct. 26 1964

Executive
Director Robert C. Stephenson Date 6 Nov. 1964

THIS FIGURE IS A LARGE MAP AND TWO SMALL PAGES WHICH ARE TO BE FOLDED AND INSERTED IN A LARGE ENVELOPE INSIDE THE BACK COVER.

Figure 12

Map of the top of Mt. Chapman

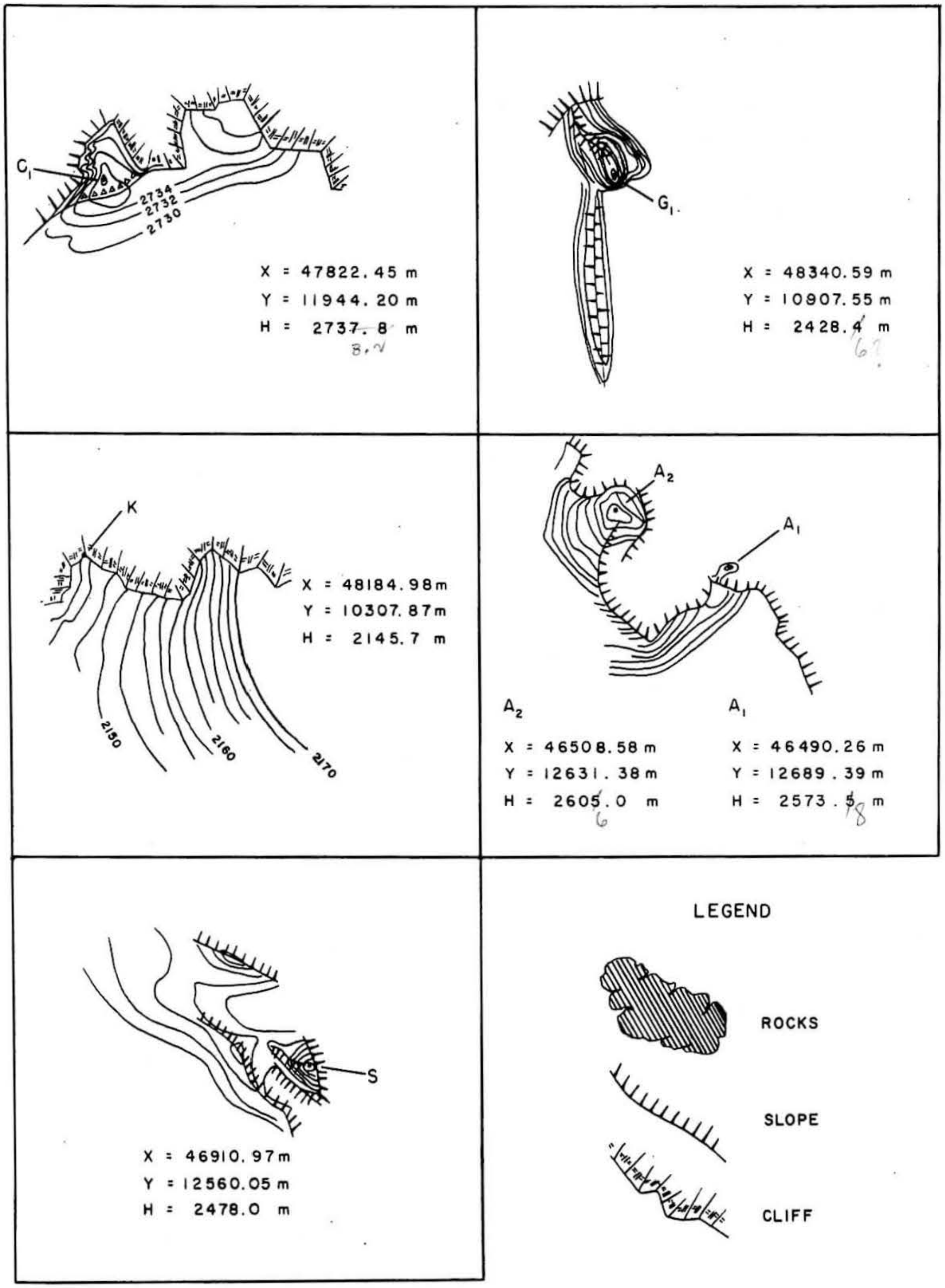
C1
2734
2732
2730
X = 47822.45 m
Y = 11944.20 m
H = 2737.8 m
G1
X = 48340.59 m
Y = 10907.55 m
H = 2428.4 m
K
X = 48184.98m
Y = 10307.87m
H = 2145.7 m
2150
2160
2170
A2
A1
A2
X = 46508.58 m
Y = 12631.38 m
H = 2605.0 m
A1
X = 46490.26 m
Y = 12689.39 m
H = 2573.5 m
S
X = 46910.97m
Y = 12560.05 m
H = 2478.0 m
LEGEND
ROCKS
SLOPE
CLIFF

Fig. 12

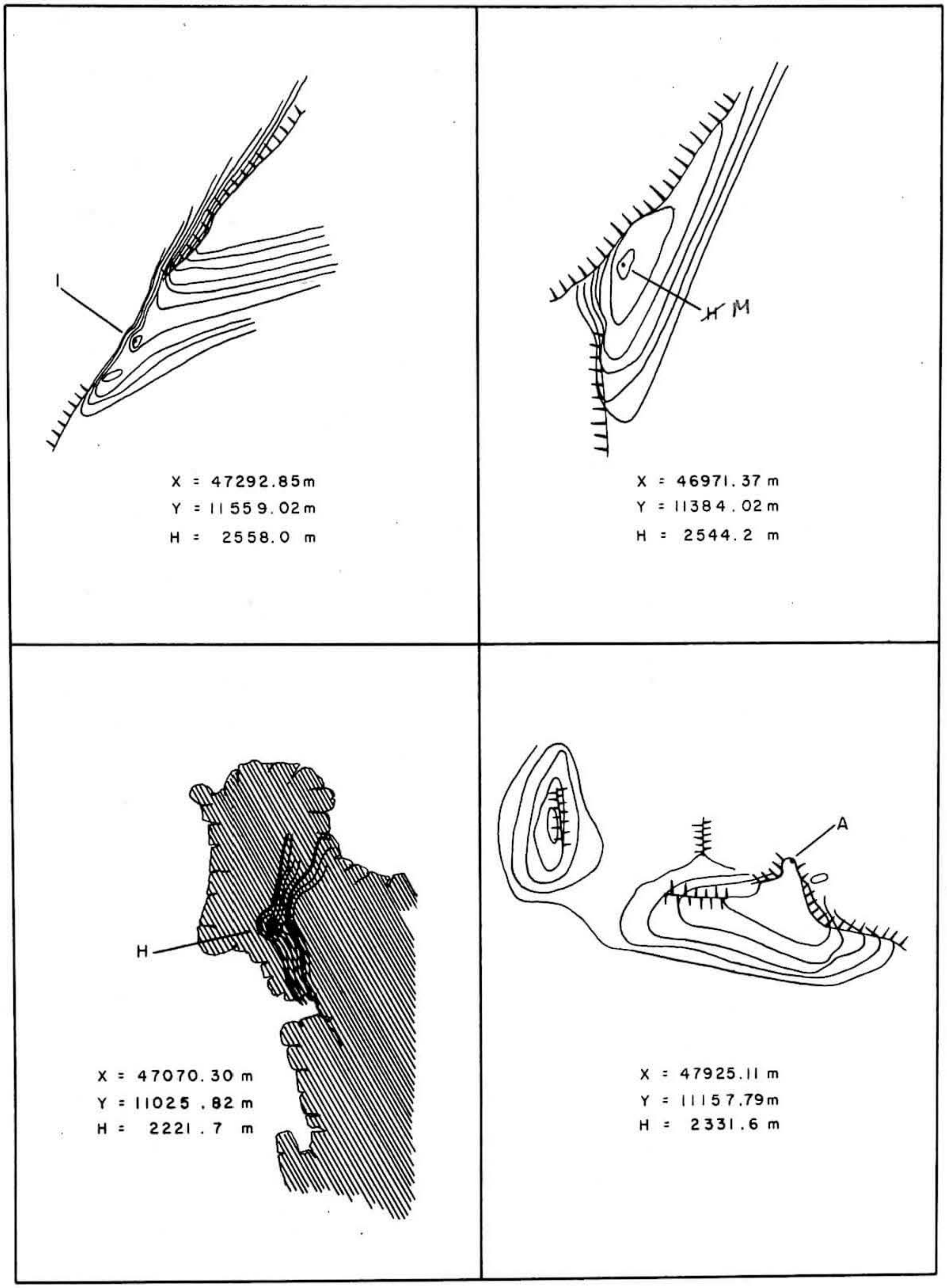
I
X = 47292.85m
Y = 11559.02m
H = 2558.0 m
H M
X = 46971.37 m
Y = 11384.02 m
H = 2544.2 m
H
X = 47070.30 m
Y = 11025 .82 m
H = 2221 .7 m
A
X = 47925.11 m
Y = 11157.79m
H = 2331.6 m

Fig. 12